PIPE DISCHARGE FLOW CALCULATIONS
(A DIERS Users Group Round-Robin Exercise)

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Review and Update

- Completed nozzle discharge flow calculations for three compositions:
  (I) Cyclohexane (10 bar)
  (II) 20% mole Ethane in Heptane (10 bar)
  (III) 2.5% mole N_2 in Cyclohexane (33 bar)
- Methods used:
  omega method
  PR-EOS flash
  ASPEN Plus & Dynamics
  SIMSCI PRO II
  SuperChem
  VENT (CISP)

Data Submittal

Include a summary sheet listing methods.

- Recommend using either $f_{TP} = 0.005$ or Re no. dependent f_{TP}.
- Use homogeneous-equilibrium model (HEM).
- Provide, P, T, x (quality) along the pipe (if available), pipe exit pressure P_{ex}, mass flux G, and discharge rate W (kg/s) corr. to flow area A_p of 3.355 in^2 (2165 mm^2).
- E-mail to Joseph Leung (DIERS UG Design/Testing Committee Chair) at leunginc@cox.net or leung@fauske.com.

Proposed Inlet Conditions

Case	Liquid Composition (mole)	P_o (bar)	T_o (°C)	x_o (vapor mass frac.)
Ia	100% c-C6	10	182.3	0.0001 (bubble pt)
Ib	100% c-C6	10	182.3	0.01
Ic	100% c-C6	10	182.3	0.1
IIa	20% C2/n-C7	10	51.9	0.0001 (bubble pt)
IIb	20% C2/n-C7	10	51.9	0.01
IIc	20% C2/n-C7	10	51.9	0.1
IIIa	2.5% N_2/c-C6	33	25	0.0001 (bubble pt)
IIIb	2.5% N_2/c-C6	33	25	0.01
IIIc	2.5% N_2/c-C6	33	25	0.1

Horizontal Pipe Discharge Problem

Same identical inlet (two-phase) conditions as the nozzle case.

- Two different piping (frictional) resistance

	Pipe I	**Pipe II**
D	2.067 in	2.067 in
L/D	50	225
L	8.61 ft	38.8 ft
K_{en}	0.5	0.5
N	1.5	5.0

Note - $N = K_{en} + 4f_{TP}\,L/D$, with $f_{TP} = 0.005$, and $K_{en} = 0.5$

Horizontal Pipe Discharge Problem - (Cont'd)

- Alternate use of Reynolds no. dependent f_{TP}

$$f_{TP} = \text{function}\left(\frac{GD}{\mu_{TP}}\right)$$

$$N = K_{en} + 4\,\bar{f}_{TP}\,\frac{L}{D}$$

where $\bar{f}_{TP}$ average two – phase friction factor

$$\mu_{TP} = \left[\frac{x}{\mu_g} + \frac{(1-x)}{\mu_f}\right]^{-1} \text{ according to McAdam}$$

μ_g, μ_f = vapor and liquid viscosity

Pipe Flow Formulation

- Constant diameter pipe (continuity) –

  $$G = \rho u = \text{constant}$$

- Energy balance (adiabatic flow) -

  $$H_1 + \frac{1}{2} G_1^2 v_1 = H_2 + \frac{1}{2} G_2^2 v_2 = \text{constant}$$

- Momentum balance (turbulent flow) -

  $$vdP + G^2 vdv + \frac{4f}{2D} G^2 v^2 dZ = 0$$

Expansion Law (Eq. of State)

- Need P-v (pressure - sp. volume) relation.

- Normal practice is to use constant H (enthalpy) flash calculation.

- From adiabatic flow starting from stagnation -

$$H_o = H + \frac{1}{2} G^2 v$$

a constant H flash assumes K.E. to be small.

Example Illustration

- Vapor cyclohexane discharge through pipe.
- Classical ideal-gas (IG) method:

  Shapiro text (1953)

  Bird Stewart Lightfoot text (1960)

  Levenspiel AIChE J (1977) – Lappel correction

  Churchill text (1980)

  Coulson & Richardson text (1996)

- Omega method.
- Constant H analytical integration method.

Classical IG Method

- $C_p / C_v = k = 1.05$

  ideal – gas specific heat values from DIPPR

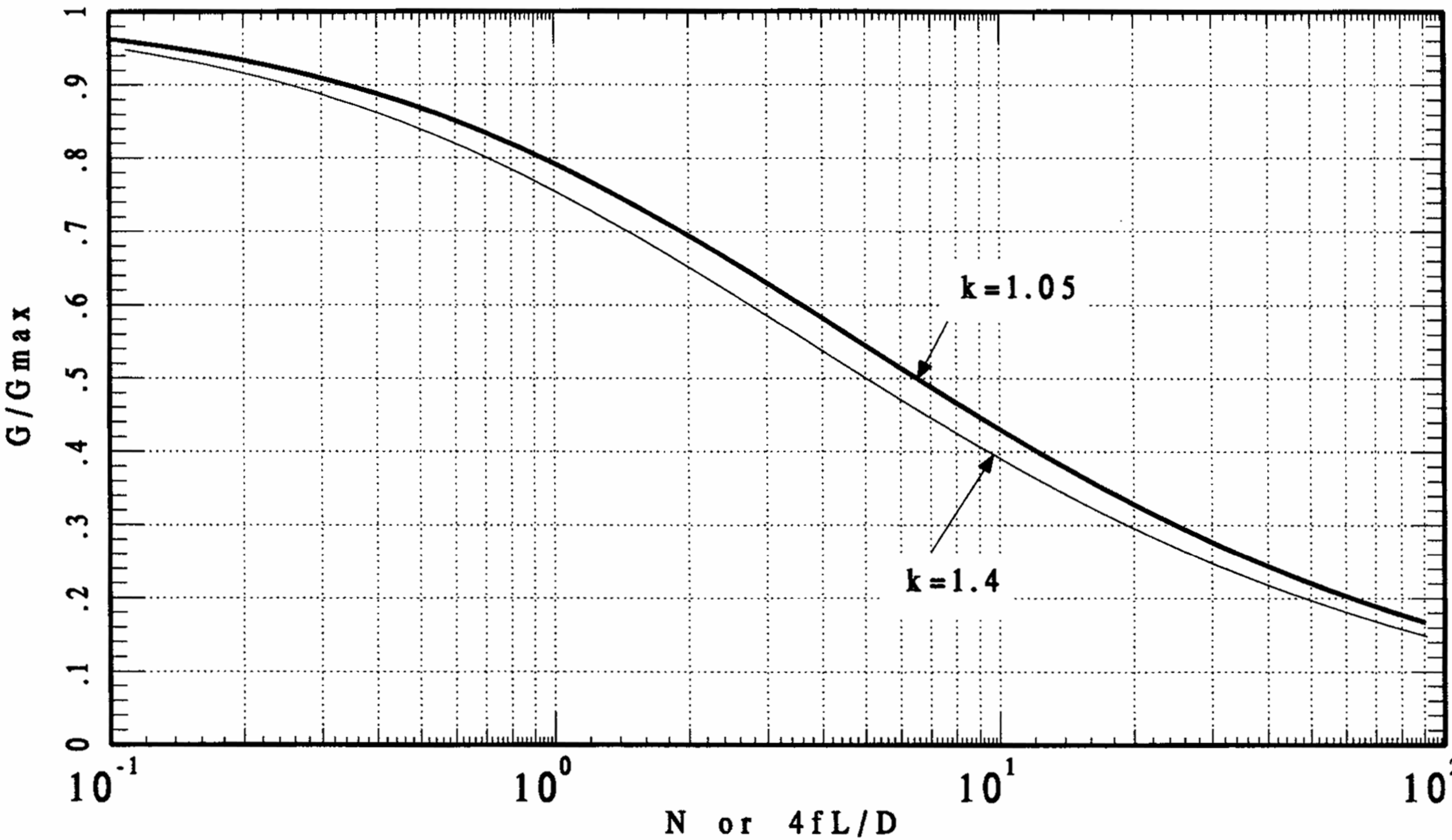
- P – v relation (exact)

$$\frac{P}{P_1} = \frac{v_1}{v}\left[1 - \left(\frac{k-1}{2k}\right)\frac{G^2 v_1}{P_1}\left(\left(\frac{v}{v_1}\right)^2 - 1\right)\right]$$

- Momentum equation

$$4f\frac{L}{D} = \frac{k+1}{k}\ln\left(\frac{v_1}{v_2}\right) + \left[1 - \left(\frac{v_1}{v_2}\right)^2\right]\left(\frac{k-1}{2K} + \frac{P_1}{G^2 v_1}\right)$$

SHAPIRO GAS PIPE FLOW CHART



Results from Classical IG Model

IG Density $\rho_{go}^{IG} = 22.3\ kg/m^3$ (10 bar, 455K)

DIPPR Density $\rho_{go} = 27.6\ kg/m^3$ $(Z_o = 0.81)$

	Pipe I	**Pipe II**
N	1.5	5.0
IG Density	2130 kg/m^2s	1560 kg/m^2s
DIPPR Density	2370 kg/m^2s	1740 kg/m^2s

Omega Method

- ω parameter at $x_o = 1.0$ is given by

$$\omega = \left(1 - 2\, P_o \frac{v_{fgo}}{h_{fgo}}\right) + \rho_{go} C_p T_o P_o \left(\frac{v_{fgo}}{h_{fgo}}\right)^2 = 1.31$$

- Momentum equation $\left(G^* = G / \sqrt{P_o \rho_o}\right)$

$$4f \frac{L}{D} = \frac{2}{G^{*2}} \left[\frac{\eta_1 - \eta_2}{1 - \omega} + \frac{\omega}{(1-\omega)^2} \ln \frac{(1-\omega)\eta_2 + \omega}{(1-\omega)\eta_1 + \omega} \right]$$

$$- 2 \ln \left[\frac{(1-\omega)\eta_2 + \omega}{(1-\omega)\eta_1 + \omega} \left(\frac{\eta_1}{\eta_2} \right) \right]$$

- Exit choking criterion

$$G_c^* = \frac{\eta_{2c}}{\sqrt{\omega}}$$

Omega Method

	Pipe I	**Pipe II**
N	1.5	5.0
G*	0.418	0.311
G	2190 kg/m^2s	1630 kg/m^2s
η_{2c}	0.478	0.357

Note: P_o = 10 bar, ρ_{go} = 27.6 kg/m^3 (DIPPR)

η_{2c} is exit choking pressure ratio

Constant H Integration Method

- Obtain P – v data from constant H FLASH calculation.

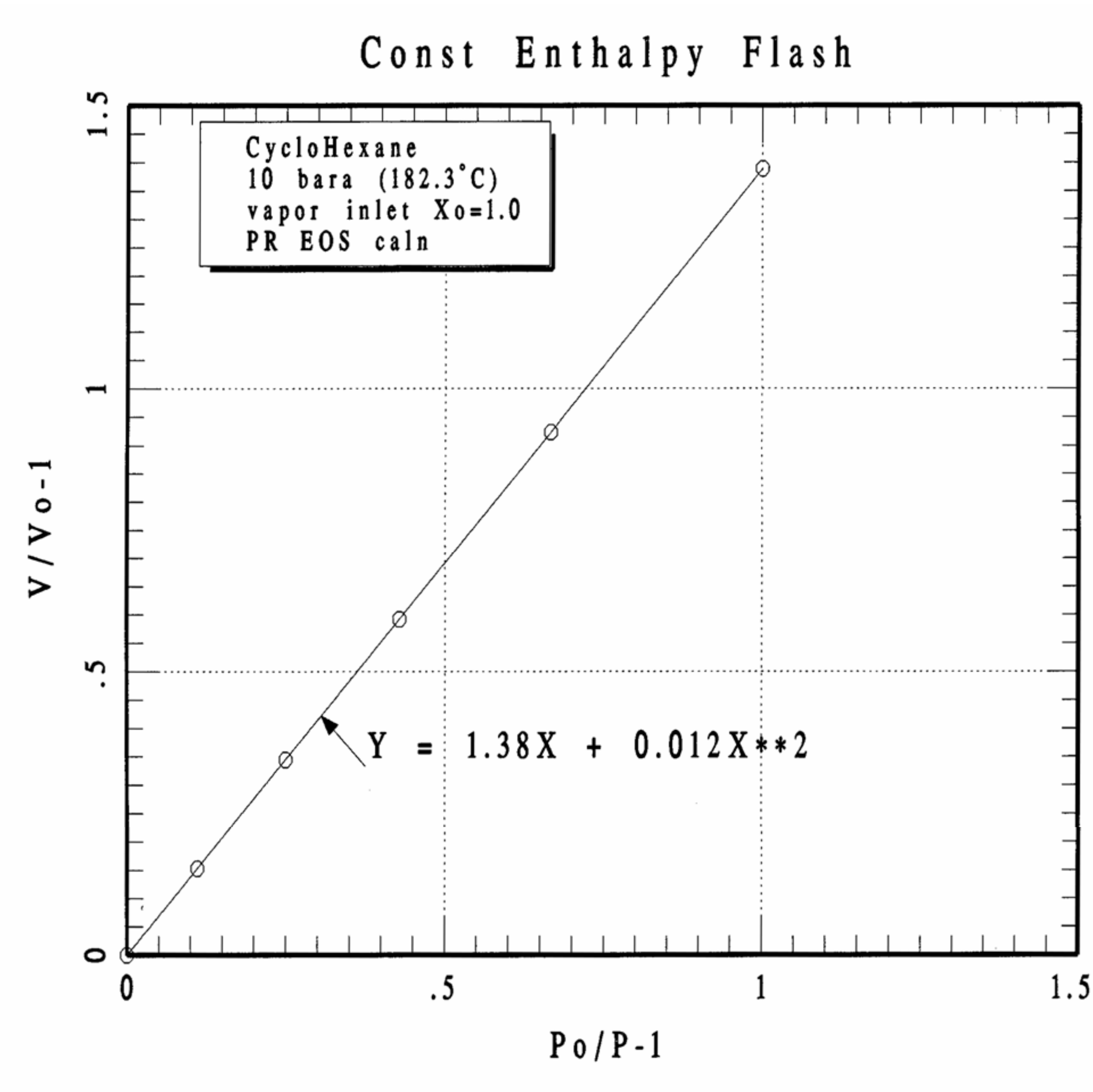
- Fit P – v with best polynomial

$$\frac{v}{v_o} - 1 = a\left(\frac{P_o}{P} - 1\right) + b\left(\frac{P_o}{P} - 1\right)^2$$

where $a = 1.38$, $b = 0.012$

- Analytical or numerical integration of differential momentum equation.

$$G^2 = \frac{2\int_{P_2}^{P_1} \frac{dP}{v}}{4f\,\frac{L}{D} + 2\ln\left(\frac{v_2}{v_1}\right)}$$


Const Enthalpy Flash
CycloHexane
10 bara (182.3°C)
vapor inlet Xo=1.0
PR EOS caln
Y = 1.38X + 0.012X**2
V/Vo-1
Po/P-1
0
.5
1
1.5
0
.5
1
1.5

Analytical Integration Method

	Pipe I	**Pipe II**
N	1.5	5.0
G*	0.412	0.307
G	2160 kg/m^2s	1610 kg/m^2s
η_{2c}	0.488	0.366

Note: P_o = 10 bar, ρ_{go} = 27.6 kg/m^3 (DIPPR)

Summary of c-C6 Vapor Discharge Rate

	Pipe I (N = 1.5)	**Pipe II (N = 5)**
IG (k = 1.05)	4.61 kg/s	3.38 kg/s
IG w/ real ρ_{go}	5.13 kg/s	3.77 kg/s
ω method	4.74 kg/s	3.53 kg/s
Const H analytical	4.68 kg/s	3.48 kg/s
Average	**4.79 kg/s**	**3.54 kg/s**
Std. Dev.	0.23 kg/s 5%	0.17 kg/s 1.5%

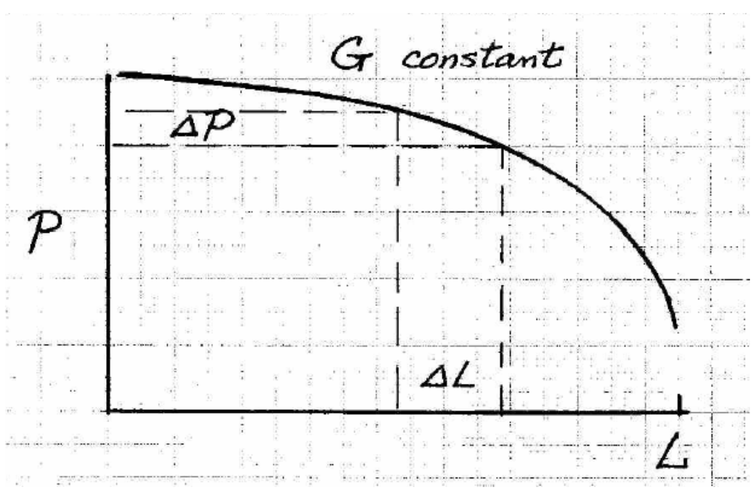
Pipe-Segment Numerical Integration

$$\Delta L = -\frac{\bar{v}\,\Delta P + G^2\,\bar{v}\,\Delta v}{\frac{2f}{D}\,G^2\,\bar{v}^2}$$

where ΔP is pressure increment

Δv is incremental specific volume over ΔP

$\bar{v}$ is average specific volume in ΔP



Numerical Integration Steps

1. G is known or guessed.
2. Increments of pressure are taken from the initial to the final pressure.
3. $\overline{v}$ and Δv are obtained for each increment for a constant-enthalpy process.
4. ΔL for each ΔP taken is computed from Eq. in previous slide.
5. Total length of pipe L is $\sum \Delta L$.
6. If ΔL is negative, then ΔP is too large.
7. A critical flow condition corresponds to $\Delta L = 0$, and the final pressure corresponds to choked pressure.
8. If $\sum \Delta L > L$, then G was guessed too small and Steps 1-7 are repeated with a larger G. If $\sum \Delta L < L$, then G was guessed too large; Steps 1-7 are repeated with a smaller G.
9. A converged solution is obtained when $\sum \Delta L = L$ to within a given tolerance.

Ref.: Perry's ChE Hdb, "Fluid Dynamics" section,7th ed., also Leung, CEP article, 1996.